

Even Split

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First, let's find what is the minimal possible length of the longest segment. Using binary search on said length suppose we need to test if len is enough. We can show that it is necessary and sufficient if we can find n segments of length len (maybe intersecting) that cover the whole of Segmentland that can be bijected to n houses so that corresponding segment contains the corresponding house. This can be done greedily going from left to right and always trying to fit the next segment in the rightmost way so that we still cover some prefix of Segmentland and the corresponding house.

Now we also need to find the maximal possible length of the shortest segment. This is done similarly with binary search, only now we want to find segments of the same length that fit into Segmentland without intersection, and in our greedy algorithm we will fit them in leftmost way.

Given these two lengths, min and max , it can be shown that we can find a subdivision of Segmentland that adheres to these limits. One way to find such a subdivision is this:

- For $i \in 0 \dots n$ let's have two parameters, $c_i \leq d_i$, which means that the corresponding dividing point lies somewhere between c_i and d_i . We calculate those from left to right starting with $c_0 = d_0 = 0$, and $c_{i+1} = \max(c_i + min, a_{i+1})$, $d_{i+1} = \min(d_i + max, a_{i+2})$ (we assume $a_{n+1} = l$).
- For $i \in 0 \dots n$ let's have ans_i as an actual boundary. We will calculate it from right to left, starting with $ans_n = l$, and on each step we will select any point from $[c_i, d_i]$ that is between min and max distance from ans_{i+1} (it is easy to see there will always be at least one such point).
- Now we have an answer — $s_i = ans_{i-1}$, $f_i = ans_i$.