

Hypercatapult Commute

Problem author and developer: Pavel Mavrin

Let's build a directed graph with edges $a_i \rightarrow b_i$. Consider a (weakly) connected component in this graph. Let the size of this component be k .

For the acyclic component, we can deliver all the passengers in $k-1$ launches, making the chain of launches in the topological order.

If the component is not acyclic, we need to make at least one cycle of launches, so we need k launches. It can be shown that if it is possible to deliver all the passengers, it is possible to do it by making one big cycle of launches. Now let's look at the first launch, say $v \rightarrow u$. If we remove this launch, it will affect only passengers with $a_i = v$. So if we remove these edges, the remaining edges must form acyclic graph. We can iterate over all vertices v and check if removing this vertex makes the component acyclic, in $O(nm)$ time.